

## Plane Filling with the Golden Rhombus and the Silver Square Rhombus

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The ratio  $1 : \frac{1 + \sqrt{5}}{2}$  is called the Golden Ratio and the ratio  $1 : \sqrt{2}$  is called the Silver Ratio. The square of the Silver Ratio is  $1:2$  and this ratio is called the Silver Square Ratio. On the other hand, a rhombus has a short diagonal and a long diagonal. If the length of these diagonals is in the Golden Ratio, the rhombus is called the Golden Rhombus and, if it is in the Silver Square Ratio, the rhombus is called the Silver Square Rhombus.

At the end of June, 2010, the author discovered that the combination of the Golden Rhombus and the Silver Square Rhombus enables the plane filling. This new finding is described in this paper.

The acute angle and the obtuse angle of the Golden Rhombus are  $63^{\circ}26'$  and  $116^{\circ}34'$ , respectively. Also, the acute angle and the obtuse angle of the Silver Square Rhombus are  $53^{\circ}08'$  and  $126^{\circ}52'$ , respectively. Accordingly, if we add one obtuse angle of the Silver Square Rhombus to two obtuse angles of the Golden Rhombus, we get  $360^{\circ}$ . Likewise, four acute angles of the Golden Rhombus and two acute angles of the Silver Square Rhombus make  $360^{\circ}$ . This means that the combination of the Golden Rhombus and the Silver Square Rhombus enables the plane filling. (Refer to Fig. 1 below.) This is a periodic plane filling.

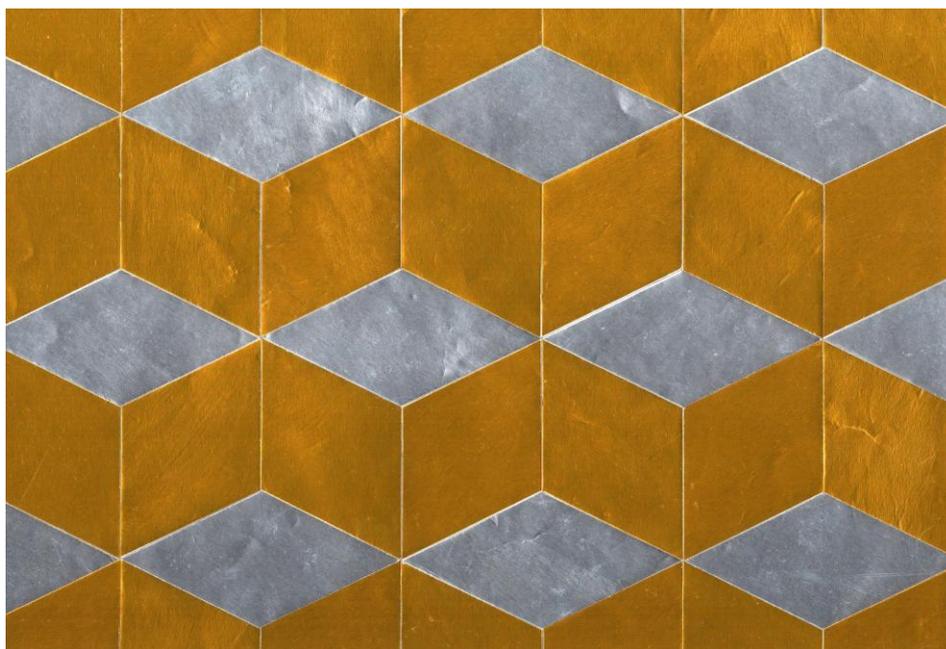


Fig. 1 Plane filling with the Golden Rhombus (in the golden color) and the Silver Square Rhombus (in the silver color)

The pattern whereby the plane is completely filled with only one kind of regular polygon is called the “regular plane filling” and there exist three kinds of such pattern in total. There also

exists a case where more than two kinds of regular polygon get together around each vertex in the same condition. It is called the “semi-regular plane filling of Archimedes” and there exist 8 kinds of such pattern in all. For instance, when a regular A polygon, a regular B polygon, and regular C polygon get together around the vertex in this order, a semi-regular plane filling symbol (A, B, C) is given.

In case of the “semi-regular plane filling”, if the center of each regular polygon is changed to the vertex, entirely new plane filling pattern is created. It is called the “dual plane filling against the semi-regular plane filling”. This plane filling is also periodic.

We now think of (3, 6, 3, 6) which is one of the semi-regular plane filling patterns. In this case, the dual plane filling against it is based on only one kind of rhombus composed of two regular triangles. The aforementioned new plane filling pattern is seemingly similar to this pattern (3, 6, 3, 6). (Refer to Fig. 2a and Fig. 2b.)

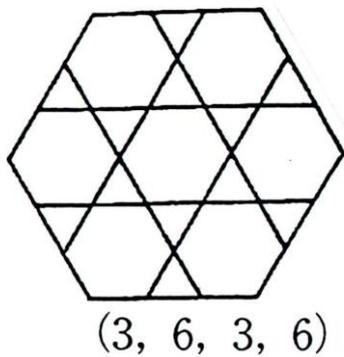


Fig. 2a Semi-regular plane filling

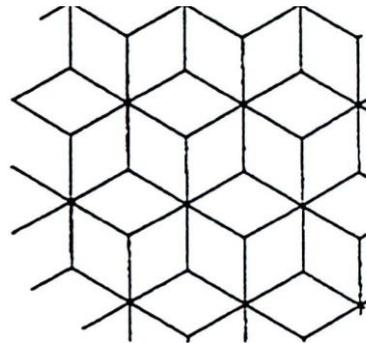


Fig. 2b Dual plane filling against (3, 6, 3, 6)

Sir Roger Penrose, a famous mathematical physicist of Oxford, discovered two kinds of non-periodical plane filling patterns in 1974. One is composed of a sagittal shape and a trapezoidal shape and the other consists of two kinds of rhombus. These two patterns are basically the same and such four quadrangles are all derived from the regular pentagon. For this reason, we can say that Penrose’s plane filling is predicated on the Golden Ratio. In comparison with it, the above-mentioned new plane filling is based on the Golden Ratio and the Silver Square Ratio.

The fact that the combination of the Golden Rhombus and the Silver Square Rhombus enables the plane filling indicates that these two ratios are closely related to each other. The mutually complementary relationship between the Golden Ratio and the silver ratio in the three-dimensional space was found by the author in 2009. If this point is taken into account, the new plane filling seems to suggest that, in the two-dimensional surface, there exists a mutually complementary relationship between the Golden Ratio and the Silver Square Ratio.

● References

- [1] A. Beutelspacher & B. Petri, "Der Goldene Schnitt", ELSEVIER GmbH, Spektrum Akademischer Verlag, 1996
- [2] K. Miyazaki, "Panorama of Shapes", Maruzen, 2003